

# Thermal entanglement properties of $N$ -qubit quantum Heisenberg chain in a two-component magnetic field

Ümit Akıncı, Erol Vatansever\*, Yusuf Yüksel

*Department of Physics, Dokuz Eylül University, Tr-35160 İzmir, Turkey*

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## Abstract

We elucidate the finite temperature entanglement properties of  $N = 9$  qubits Heisenberg XX and XXZ models under the presence of a polarized magnetic field in  $xz$  plane by means of concurrence concept. We perform a systematic analysis for a wide range of the system parameters. Our results suggest that the global phase regions which separate the entangled and non-entangled regions sensitively depend upon the spin-spin interaction term of the  $z$ - component of two neighboring spins  $J_z/J_x$ , temperature as well as polarized magnetic field components. Thereby, we think that polarized magnetic field can be used a control parameter to determine the amount of thermal entanglement between pair of qubits for different temperatures and spin-spin interaction terms. Moreover, it has been found that the nearest-neighbor pair of qubits does not point out a re-entrant type entanglement character when one only deals with the nearest-neighbor pair of qubits. However, as one considers next-nearest neighbor pair of qubits, it is possible to see the evidences of re-entrant type entanglement behaviors.

**Keywords:** Thermal entanglement, Quantum correlations,  $N$ - qubit Heisenberg XX and XXZ models, Polarized magnetic field.

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## 1. Introduction

When the nonlocal quantum correlations become important in a many-level system, one may not extract the complete information about individual sub-levels although the total information about the whole system is known. In such a case, sub-levels become strongly correlated with each other, and this fact may allow the information between two distant points (such as two qubits separated by large distances) to be communicated instantly. This phenomenon is called action at a distance, and such a pair of sub-levels is called entangled. Formerly, Einstein and his co-authors [1], as well as Schrödinger [2, 3] argued that this “spooky action at a distance” is a direct consequence of incomplete nature of quantum mechanics. However, after three decades, Bell [4] showed that Einstein’s realist idea based on the locality is wrong and it pioneered consecutive experimental realizations which proved that the predictions of quantum mechanics are true regarding the entanglement phenomenon as a nonlocal property of nature.

During the last two decades, a great many experimental efforts have been devoted to entanglement phenomenon in a wide variety of physical systems including entanglement of many photons, mesoscopic systems, and so on [5]. Hensen et al. [6] very recently produced 245 entangled pairs of electrons (which were 1.3 kilometers apart from each other) in nine days. They reported that their results rule out large classes of local realist theories. On the other hand, in the theoretical ground, it was quite a challenge to measure the amount of entanglement between two correlated sub-systems [7, 8, 9, 10, 11]. The two distinct measures to distinguish between entangled and separable states are concurrence [12] and negativity [13]. One should notice that concurrence cannot be used as a criterion for separability condition for the systems with dimensions larger than  $2 \otimes 3$  in Hilbert space. Using concurrence as a measure of entanglement between two qubits, models based on localized spins in the form of Ising, XY and isotropic, as well as anisotropic Heisenberg systems have been widely investigated in the literature [14, 15, 16, 17, 18, 19, 20, 21, 22]. In

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\*Corresponding author. Tel.: +90 3019547; fax: +90 2324534188.

Email address: erol.vatansever@deu.edu.tr (Erol Vatansever\*)

order to observe entanglement phenomenon in such systems, selected Hamiltonian should include either off-diagonal terms such as anisotropic exchange coupling and Dzyaloshinskii-Moriya (DM) interaction, and/or inhomogeneous external magnetic fields along the Ising axis.

Apart from these, pairwise entanglement in the systems with three or more qubits [23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45] have also been studied in the forms of XX, XY and Heisenberg models, as well as in the forms of their varieties. According to these works, some important findings can be listed as follows: Under certain conditions, next-nearest-neighbor entanglement may be larger than nearest-neighbor entanglement near zero temperature [24]. As the number of qubits becomes larger than a larger value of external homogeneous magnetic field is needed to observe entanglement. However, entanglement disappears shortly after the field exceeds some critical value [25]. Moreover, isotropic Heisenberg spin chain exhibits non-zero concurrence only when the exchange coupling is of antiferromagnetic type [26, 27] whereas if one applies a magnetic field then the SU(2) symmetry is broken and it becomes possible for a ferromagnetic isotropic Heisenberg chain to have thermal and ground states which are completely entangled [28]. For XX qubit rings with periodic boundary conditions, Ref.[29] also realized that pairwise entanglement between the nearest-neighbor qubits is invariant under the magnetic field reversal  $B \rightarrow -B$ , and that for the same model containing “even number of qubits”, bipartite thermal entanglement between neighboring sites should be independent of both the sign of magnetic fields and exchange constants. Ref. [45] showed for isotropic Heisenberg model that the ground state entanglement becomes enhanced (diminished) with increasing number of qubits in odd (even)-numbered qubit rings. It is also possible to distinguish between thermally entangled and separable states via examining macroscopic properties such as specific heat and magnetic susceptibility which can play the role of some kind of entanglement witness [46].

There are also some other works dealing with entanglement properties of qubit-qutrit and qutrit-qutrit chains [47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59]. In an extended work, Wang et al. [60] studied the entanglement in a spin-1/2 and spin- $s$  ferrimagnetic chain in which they reported that as the magnitude of spin- $s$  increases then the temperature value at which entanglement vanishes becomes higher whereas the ground state thermal entanglement for small- $s$  chains is enhanced. Similarly, Ref. [61] showed that threshold temperature at which entanglement vanishes increases with increasing spin magnitude.

In practice, it is a difficult task to control the entanglement in a system by manipulating the exchange interactions, and in some cases, the control of the magnitude and direction of externally applied magnetic field proved useful for tuning the entanglement of a spin chain [62, 63, 64, 65]. Therefore, in the present paper, our aim is to clarify the entanglement phenomena in  $N$ - qubit XX and XXZ chains in the presence of magnetic fields applied in both longitudinal (i.e. easy axis) and transverse (hard axis) directions. The outline of the paper can be summarized as follows: In Sec. 2 we define our model. Numerical results are presented in Sec. 3. Finally Sec. 4 contains our conclusions.

## 2. Formulation

We consider 1D Heisenberg XX and XXZ spin chain systems consisting of  $N$ - spin-1/2 particles interacting with nearest neighbor interaction. Each qubit in the system is under the influence of a polarized magnetic field applied in  $xz$  plane. Within the open boundary condition, the Hamiltonian of a such system can be described as follows:

$$\mathcal{H} = \sum_{i=1}^{N-1} \left[ J_x (\sigma_i^x \sigma_{i+1}^x + \sigma_i^y \sigma_{i+1}^y) + J_z \sigma_i^z \sigma_{i+1}^z \right] + h_x \sum_{i=1}^N \sigma_i^x + h_z \sum_{i=1}^N \sigma_i^z \quad (1)$$

where  $\sigma_i^x$ ,  $\sigma_i^y$  and  $\sigma_i^z$  are the Pauli spin operators at the site  $i$ .  $J_x$  and  $J_z$  are spin-spin exchange interaction terms of the  $x$ - and  $z$ - components of two neighboring spins which are selected to be as  $J_x > 0$  and  $J_z > 0$ , i.e., antiferromagnetic type interaction. Throughout the work, we have fixed the total number of particle  $N = 9$  and value of spin-spin interaction  $J_x = 1$ .  $h_x = h_0 \cos \theta$  and  $h_z = h_0 \sin \theta$  are the components of the external polarized field. Here,  $h_0$  and  $\theta$  are the amplitude of the field and the angle between the field and its corresponding component of the Pauli spin operators, respectively. In Eq. (1), the first summation is over the nearest neighbor qubits while the second and third ones are over all of the qubits. The Hamiltonian considered here covers XX model with  $J_z = 0$  and isotropic Heisenberg model with  $J_z = 1$  in the limit cases. There are several studies in the literature regarding the limit cases and some variants of the model Hamiltonian [26, 27, 28, 29, 30, 32, 36, 37, 38, 39, 42, 44, 61, 66, 67, 68, 69, 70].

The physical properties of the system in thermal equilibrium can be obtained via the density matrix operator, which can be defined as follows:

$$\rho = \frac{1}{Z} \exp(-\beta \mathcal{H}) \quad (2)$$

here  $\beta = 1/k_B T$ ,  $T$  and  $k_B$  are the temperature and Boltzmann constant, respectively.  $Z$  is the partition function which is defined by the trace of the density matrix as  $Z = \text{Tr} \exp(-\beta \mathcal{H})$ . The basic strategy for obtaining the concurrence (as a measure of bipartite entanglement) for two qubits (say  $\sigma_m$  and  $\sigma_n$ ) is, constructing the reduced density matrix by tracing out the other spins, and calculating  $C_{mn}$  via [12]

$$C_{mn}(R) = \max \left\{ 2\lambda_1 - \sum_{i=1}^4 \lambda_i, 0 \right\} \quad (3)$$

where  $\lambda_i$ 's are the square roots of the eigenvalues of the operator  $R$  which is obtained by

$$R = \rho^{(mm)} (\sigma_m^y \times \sigma_n^y) \rho^{(nn)*} (\sigma_m^y \times \sigma_n^y) \quad (4)$$

in descending order. Here,  $\sigma_m^y$  is the  $y$ - component of the Pauli spin matrix related to the spin  $m$ ,  $\rho^{mm}$  is reduced density matrix, and  $*$  denotes the complex conjugation. The value of concurrence varies from  $C = 0$  for a separable state (no entanglement) to  $C = 1$  for a maximally entangled state.

### 3. Results and Discussion

In this section, we will focus our attention on the thermal entanglement properties of the  $N = 9$  qubits  $XX$  and  $XXZ$  Heisenberg spin chains under the existence of a polarized magnetic field. We especially study the pairwise entanglement between central qubit and its nearest neighbor entanglement  $C_{56}$  and as well as next-nearest entanglement  $C_{57}$  to show how the applied magnetic fields, interaction constant of the  $z$ - component and the temperature affect the natures of entanglement features of the system. For this aim, the boundaries in related planes separating entangled and non-entangled regions will be given for the selected Hamiltonian parameters. The physical mechanisms underlying of these types of observations will be discussed in detail. Depending on the considered Hamiltonian parameters, in the entangled region, the concurrence has a non-zero value corresponding to the presence of a strongly correlated two qubit pair. Contrary to this, concurrence vanishes in the non-entangled region. Keeping these facts in mind, we illustrate the boundaries in the  $(h_x/J_x - h_z/J_x)$  contour map in Figs. 1(a-d) with varying  $J_z/J_x$  parameters such as  $J_z/J_x = 0.0$  (a),  $0.5$  (b),  $1.0$  (c) and  $1.5$  (d), respectively. The figures are presented for the nearest-neighbor concurrence  $C_{56}$ , and for the relatively low value of temperature, i.e. for  $k_B T/J_x = 0.1$ . It is clear from figure 1(a) that along the  $h_x/J_x = 0.0$  axis, when the value of  $|h_z/J_x|$  increases starting from zero, the concurrence  $C_{56}$  tends to rise until it reaches a certain value for a given temperature. Then,  $C_{56}$  begins to decrease with further increment in value of  $|h_z/J_x|$ . Moreover, as the value of  $|h_z/J_x|$  is sufficiently large, the entanglement between qubits 5 and 6 disappears, hence,  $C_{56}$  vanishes. These behaviors appear symmetrically at two sides of  $h_z/J_x$ . Along the  $h_z/J_x = 0.0$  axis, magnetic field dependency ( $h_x/J_x$ ) of  $C_{56}$  exhibits a nearly similar character to the previous one mentioned above. The only difference is that the entanglement between qubits 5 and 6 still lives as the value of  $|h_x/J_x|$  is sufficiently too large. Figures 1(b-d) show effect of the interaction constant  $J_z/J_x$  on the thermal entanglement character of  $C_{56}$ . Based on our numerical results, it has been found that  $J_z/J_x$  parameter plays an important role in enhancing the entanglement. As the value of  $J_z/J_x$  is too large, much more energy originating from magnetic fields is needed to drive the system from entangled state to non-entangled one. The aforementioned situations can be easily seen by comparing the figures 1(a-d) with each other. The findings also demonstrate that the geometrical shape where the entanglement character exists sensitively depends on the selected  $J_z/J_x$  parameter. For instance, figures 1(a) and (b) present an elliptical shape whereas the figure (c) shows a circular shape entanglement character. However, as depicted in figure 1(d), the general trend of entanglement character again becomes elliptical with further increment in value of  $J_z/J_x$ .

In figure 2(a-d), we examine thermal entanglement features of the next-nearest neighbor concurrence  $C_{57}$  of nine-qubit system for the same Hamiltonian parameters with figure (1). In contrary to the figure 1, in the absence of polarized magnetic field, there is no entanglement behavior between qubits 5 and 7. However, increment in strength

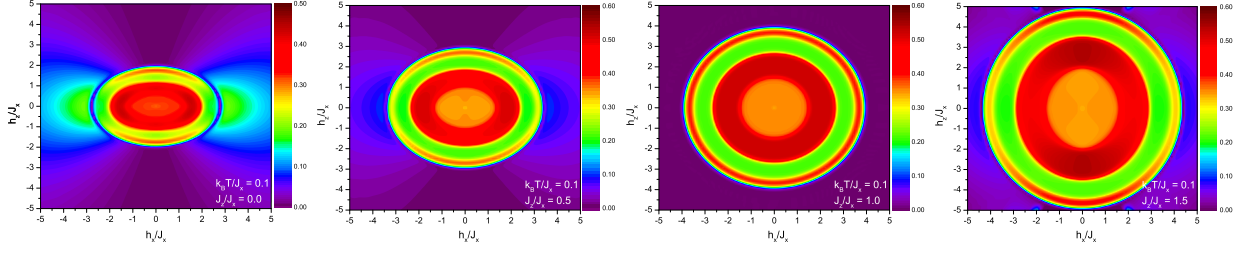


Figure 1: Contour maps of magnetic field dependencies of the nearest-neighbor concurrence  $C_{56}$  for varying values of the  $J_z/J_x$  ratio such as (a)  $J_z/J_x = 0.0$ , (b) 0.5, (c) 1.0 and (d) 1.5, respectively. The figures are plotted for the value of  $k_B T/J_x = 0.1$ .

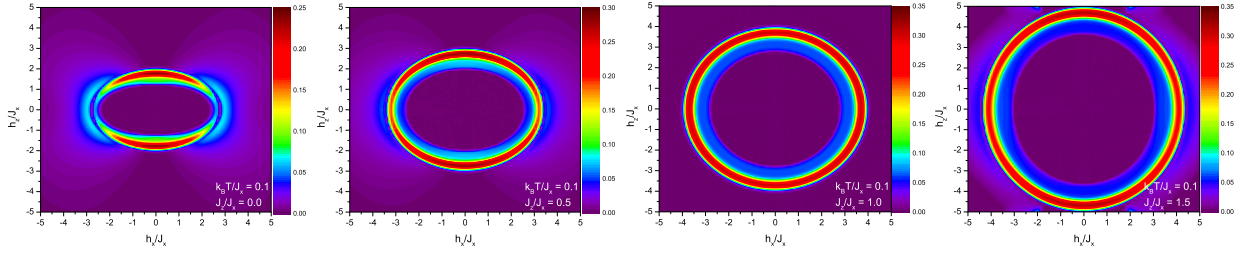


Figure 2: Contour maps of magnetic field dependencies of the next nearest-neighbor concurrence  $C_{57}$  for the same Hamiltonian parameters with figure (1).

of the magnetic field gives rise to the existence of an entanglement character. Next, as the strength of applied magnetic field is increased further, 5 and 7 qubits become non-entangled. The mechanism briefly mentioned here is often called the re-entrant type entanglement behavior. We should indicate that there are several studies in the literature showing a re-entrant type entanglement character [19, 23, 32].

In order to elucidate the temperature influences on the entanglement properties of the studied system, we plot figures (3) and (4) corresponding to the concurrences  $C_{56}$  and  $C_{57}$ , respectively. The figures are demonstrated for the value of  $k_B T/J_x = 0.5$ . The raising temperature shows a tendency to reduce and destruct the quantum correlations between the considered qubits for the fixed sets of Hamiltonian parameters. If one compares the figures (1) and (3) with each other, it can be easily seen that the concurrence  $C_{56}$  prominently decreases when the value of temperature is increased because the thermal energy is dominant against the spin-spin interactions. We should also note that although an increment in the value of temperature causes a quantitative change in concurrence  $C_{56}$ , it does not lead to a change in characteristic behavior. It is beneficial to notice that similar type observations originating from the thermal agitations have been reported in Ref. [14] where ground state and finite temperature features of two qubit system under the influence of a polarized magnetic field have been discussed in detail. Furthermore, by comparing the figures (2) and (4) with each other, it is possible to say that when the temperature increases from  $k_B T/J_x = 0.1$  to 0.5, unusual and interesting thermal entanglement behaviors occur in next-nearest neighbor concurrence  $C_{57}$ . These dramatic changes have emerged at the relatively low values of  $J_z/J_x$  such as for  $J_z/J_x = 0.0$  and 0.5. For example, non-entangled region expands in  $h_x/J_x$  and  $h_z/J_x$  plane as well as a re-entrant type entanglement character takes place along the  $h_x/J_x = 0$  lines. It means that the considered qubits are dragged from non-entangled region to entangled region with increasing value of magnetic field  $h_z/J_x$ . Then, as the energy arising from the magnetic field is increased further, the system displays opposite behavior than the previous one discussed above.

#### 4. Conclusion

In conclusion, we have investigated the thermal entanglement properties of  $N = 9$  qubits XX and XXZ Heisenberg spin chains within the open boundary condition by making use of concurrence concept. We design the system such that each qubit in the chain is exposed to the polarized magnetic field in  $xz$  plane. According to our detailed investigation,

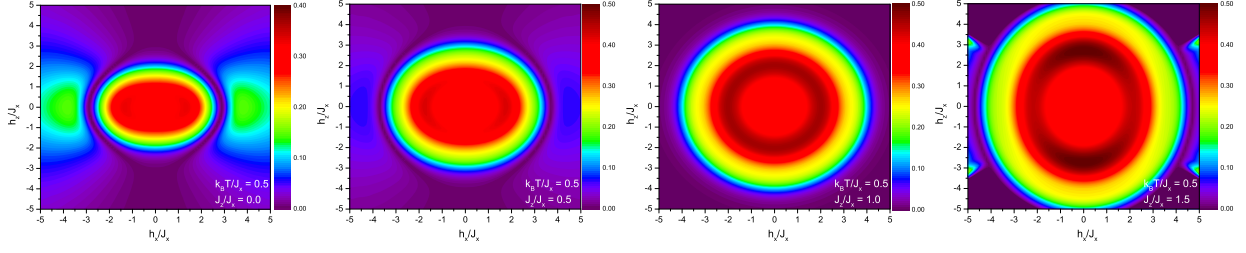


Figure 3: Contour maps of magnetic field dependencies of the nearest-neighbor concurrence  $C_{56}$  for varying values of the  $J_z/J_x$  ratio such as (a)  $J_z/J_x = 0.0$ , (b)  $0.5$ , (c)  $1.0$  and (d)  $1.5$ , respectively. The figures are plotted for the value of  $k_B T/J_x = 0.5$ .

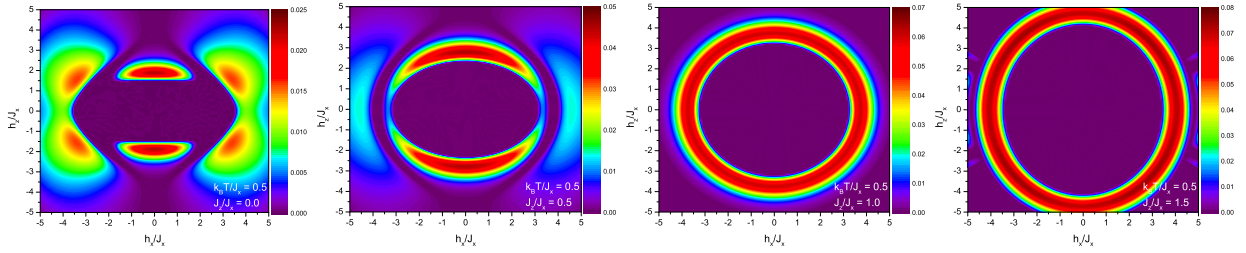


Figure 4: Contour maps of magnetic field dependencies of the next nearest-neighbor concurrence  $C_{57}$  for the same Hamiltonian parameters with figure (3).

the most conspicuous findings mentioned in the present study can be listed as follows. The boundaries which separate the entangled and non-entangled regions sensitively depend upon the spin-spin interaction term of the  $z$ - component of two neighboring spins  $J_z/J_x$ , temperature as well as polarized magnetic field components. For example,  $J_z/J_x$  parameter plays a crucial role in improving the entanglement depending on the other Hamiltonian parameters. It allows us to control the amount of the entanglement between the selected pair of qubits by varying the value of  $J_z/J_x$ . Moreover, it is possible to realize a magnetically stimulated entanglement behavior by selecting the suitable system parameters. By changing the value of polarized magnetic field, one may create or destruct the quantum correlations between the pair of qubits. It has also been found that the present system presents a re-entrant type thermal entanglement character depending on the considered system parameters if one considers the next-nearest neighbor pair of qubits. Finally, we deal with the finite temperature influences on the entanglement character of system, and the raising temperature demonstrate a tendency to shrink the quantum correlations between the considered qubits for the fixed sets of Hamiltonian parameters.

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